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ON VORTEX BREAKDOWN AND INSTABILITY

S. N. SINGH and W. L. HANKEY

Southeastern Center for
Electrical Engineering Education
St. Cloud, Florida 32769

March 1981

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A literature survey on the vortex breakdown phenomenon is presented. Several theoretical and numerical models which have been proposed and applied to explain the experimental observations concerning vortex bursting are reviewed. Required parameters to perform the related stability analysis have been identified. Appropriate criteria where possible are cited to assist engineers in estimating occurrence of vortex bursting. A numerical scheme to compute and verify some of the results for incompressible inviscid unstable

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modes is programmed on the computer at WPAFB. It is suggested that the influence of adverse pressure gradient on the vortex breakdown be investigated in detail and results thus obtained be compared with experiments under appropriate conditions.

FOREWORD

This report is the result of work carried on in the Computational Aerodynamics Group, Aerodynamics and Airframe Branch, Aeromechanics Division, Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories. It was performed by Dr. Shiva N. Singh, summer SCEEE Research Associate and directed by Dr. Wilbur L. Hankey under Contract No. F49620-79-C-0038. Dr. Singh performed the work from May 18, 1980 through July 25, 1980, while employed at the University of Kentucky, Lexington, Kentucky 40506.

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INTRODUCTION

The problem of vortex breakdown occurring over delta wings at large angles of attack and in axisymmetric swirling flows in circular pipes has recently received considerable interest. A number of review articles (Hall[17,19] and Leibovich[31]) on the subject have been written and a symposium on concentrated vortex motions in fluids (Küchemann[27]) has been arranged.

Peckham and Atkinson[40] were the first to discover the occurrence of vortex breakdown over delta wings with highly swept leading edges, when such wings were set at large angles of incidence. The work of Elle[10], Cox[8], Werle[52], Lambourne and Bryer[28], Hummel[23], Earnshaw[9], and Lowson[34] helped to construct a detailed picture of the flow field near the breakdown, as shown in Fig 1. When a slender wing is set at an angle of attack, the flow on the upper surfaces separates from near the leading edges, forming two shear layers. These layers curve upward and inboard and eventually roll up into a core of high vorticity. Most vortex cores have an appreciable axial component of motion and the fluid spirals around and along the axis. In the core of the leading edge vortex over a delta wing, velocities two to three times that of the undisturbed stream have been found. An increase in the angle of attack strengthens the vorticies and eventually there is an abrupt change in the structure of the vortex with a very pronounced retardation of the flow along the axis, followed by reversed flow in a region of limited axial extent. This abrupt change is called "vortex breakdown" or "vortex bursting."

Harvey[20] initiated the study of vortex breakdown through a long cylindrical tube. By varying the amount of swirl that was imparted to the fluid before it entered the tube, he found that the breakdown was the intermediate stage between two basic types of rotating flows, those that do and those that do not exhibit axial velocity reversal. Since then, Kirkpatrick[24], Chanaud[7], Sarpkaya[42,43,44], Faler and Leibovich[11,12] and Garg[13] performed the more easily controllable experiments in tubes and presented vast data on vortex breakdown. Two forms of vortex breakdown predominate, one called "axisymmetric" or "bubble-like" and the other called "spiral." The type and the shape of the forms depend upon the particular combination of the Reynolds and circulation numbers (see Fig. 2 for details).

Many analytical and numerical solutions of the Navier-Stokes equations have been attempted to explain the occurrence of vortex breakdown phenomenon. Hall[18] and Stewartson and Hall[48] attempted to solve the Navier-Stokes equations by proposing a simplified model for the vortex core formed over a slender delta wing at incidence by the rolling-up of the shear layer that separates from a leading edge. The incompressible quasi-cylindrical boundary-layer approximate momentum integral equations describing the flow in the viscous core of a wing-tip vortex were solved by Gartshore[14,15], Steiger and Bloom[47] and Mager[37]. Benjamin[3,4] suggested that the inviscid vortex breakdown, like a hydraulic jump is a transition between two conjugate states of flow. Bossel[5,6] analyzed the vortex breakdown flowfield by reducing the equations of motion to simpler sets in four different regions. Solutions of the steady axisymmetric Navier-Stokes equation for vortex breakdown in a confined as well as unconfined viscous vortex have been obtained by Hall[18], Kopecky and Torrance[26] and Grabowski and Berger[16].

Many authors believe that the vortex bursting with a local stagnation of the axial flow is a direct consequence of hydrodynamic instability with respect to axisymmetric, non-axisymmetric or antisymmetric infinitesimal disturbances. Ludweig[35,36] initiated the study of linear hydrodynamic stability concerning swirling flows. Then Leibovich[30,31], Randall and Leibovich[41], Uberoi, Chow and Narain[51], Lessen, Singh and Paillet[32], Lessen and Pailett[33], and Singh and Uberoi[45] carried out detailed stability analysis on the vortex breakdown.

In the forementioned research work concerning various explanations and interpretations of vortex breakdown, there is considerable overlap between theoretical predictions and experimental observations. In the following sections, the mathematical formulation of the vortex flows in cylindrical polar coordinates (r, θ, z) is given. Solution for the axisymmetric axial flow in trailing line vortices is obtained. This is imposed as the starting condition at $z = 0$ to obtain the solution profiles of full Navier-Stokes equations for $z > 0$. In order to study the linear stability analysis the mean flow is also taken as that of trailing line vortices. Theoretical solutions thus obtained are compared with experimental results and discrepancies pointed out.

SECTION II

MATHEMATICAL FORMULATION

Cylindrical polar coordinate system (r, θ, z) are used. The radial, circumferential and axial components of velocity are denoted by u, v and w respectively. The unsteady Navier-Stokes equations for incompressible medium are:

$$\frac{1}{r}\frac{\partial ur}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = \frac{-1}{\rho}\frac{\partial p}{\partial r} + \nu(\nabla^2 u - \frac{u}{r^2} - (\frac{2}{r^2})\frac{\partial v}{\partial \theta}) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{-1}{\rho}\frac{\partial p}{r\partial \theta} + \nu(\nabla^2 v - \frac{v}{r^2} + (\frac{2}{r^2})\frac{\partial u}{\partial \theta}) \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w, \tag{4}$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

and p, ρ and ν are the pressure, density and kinematic viscosity of the fluid.

Fluid flowing past a lifting wing produces trailing vorticity and this at some distance downstream, eventually concentrates into two trailing line vortices. A characteristic feature of a steady trailing line vortex is the existence of strong axial currents near the axis of symmetry. The link between the azimuthal and the axial components of motion in vorticies is provided by the pressure; the radial pressure gradient balances the centrifugal force, and any change in the azimuthal motion in the axial direction downstream produces an axial pressure gradient and consequently axial acceleration (see Squire[46], Newmann[38], Batchelor[1], Owen[29] and Uberoi[50] for discussion on trailing vorticies).

To derive the expression for the velocity components in the case of trailing line vortices, flow fields are assumed to be steady and axisymmetric, such that the axial variations are smaller than the radial derivatives, i.e.

$$\frac{\partial}{\partial z} << \frac{\partial}{\partial r}, \qquad w >> u . \tag{5}$$

We further approximate that the axial velocity w is nearly equal to the free stream velocity W, then equations (1) to (4) reduce to

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{6}$$

$$-\frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{7}$$

$$W\frac{\partial v}{\partial z} = \nu\left(\frac{\partial^2 v}{\partial r^2}\right) + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} \tag{8}$$

$$W\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{9}$$

A new independent variable ζ is introduced in place of r and is defined as

$$\zeta = Wr^2/4\nu z \tag{10}$$

Solution under appropriate conditions is obtained as (see Batchelor[1] and certain comments by Tam[49] and Herron[21])

$$v = \frac{c_o}{r}\left(1 - e^{-\zeta}\right), \tag{11}$$

$$w = W - \left(\frac{c_o^2}{8\nu z} \operatorname{Log} \frac{Wz}{\nu}\right) e^{-\zeta} + \frac{c_o^2}{8\nu z} f(\zeta) - \frac{LW^2}{8\nu z} e^{-\zeta} \tag{12}$$

$$\frac{Po - P}{\rho} = \frac{c_o^2 W}{8\nu z}\left[\frac{(1 - e^{-\zeta})^2}{\zeta} + 2\, ei(\zeta) - 2ei(2\zeta)\right], \tag{13}$$

where

$$f(\zeta) = e^{-\zeta}\{\log \zeta + ei(\zeta) - 0.807\} + 2ei(\zeta) - 2ei(2\zeta),$$

and $$ei(\zeta) = \int_\zeta^\infty \frac{e^{-\xi}}{\xi}\, d\xi .$$

c_o is the constant circulation at large radius r and L is a constant depending on the induced drag or the initial velocity defect in the presimilarity stage. Uberoi[50] has shown that the expressions (11) to (13) for v, w and p neglect the radial and associated axial convection of angular momentum and the radial velocity component is assumed negligibly small. As a result the trailing vortex is reduced to a line vortex and thus the approximation may be invalid. Although the solution represented by (11-13) may not be quite accurate, it is generally adequate for many purposes. Further studies have assumed this solution as the starting condition at $z = 0$ to calculate the subsequent development of the vortex breakdown. Also, the experimental measurements of the velocity distribution inside a swirling tube are quite close to those given by (11) and (12) (see Garg[13]).

SECTION III

SOLUTIONS OF THE NAVIER-STOKES EQUATIONS

In non-viscous flows, in addition to theorems of Bernoulli and Helmholtz-Kelvin, Crocco[17] developed an interesting result concerning vortex flows. Euler's equation for inviscid compressible gases can be written as

$$(\partial\vec{u}/\partial t) - \vec{u}\times(\nabla\times\vec{u}) + \nabla(\vec{u}\cdot\vec{u}/2) = -\nabla h + T\nabla s \tag{14}$$

where $\vec{u}$ is the velocity vector, and h, T and s are the enthalpy, temperature and entropy respectively. Making use of Bernoulli theorem equation (14) reduces to (for steady state)

$$\vec{u}\times(\nabla\times\vec{u}) = -T(\nabla s) \tag{15}$$

Two conclusions can be drawn from (15):

1) An irrotational stationary gas flow is an isentropic motion;

2) If there are entropy changes within the field, vortices (vortex sheets) will generally appear. Or the vortex flows are non-isentropic, unless the velocity and vorticity vectors are parallel to each other, a very special case.

On the basis of inviscid theory, Squire[46], Benjamin[3,4], and Bossel[5] considered the following differential equation (see Squire and Benjamin for its derivation)

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = r^2\frac{dH}{d\psi} - k\frac{\partial k}{d\psi} \tag{16}$$

where ψ is the Stokes stream function. H is the total head which by Bernoulli's theorem is a function of ψ alone and k = rv, the circulation about the z-axis is also a function of ψ alone according to Kelvin's theorem. The above-mentioned authors attempted to obtain solution of (16) in terms of axisymmetric waves. They regarded the vortex breakdown phenomenon as the existence of a critical state. Vortex flows have been classified as 'supercritical' if the wave can propagate with phase speeds in the downward direction ($c > 0$) or subcritical if the propagation is also possible in the upstream direction ($c < 0$). 'Critical' flow separates the two classes where $c = 0$. Benjamin proposed that vortex breakdown can be explained as a transition between two (supercritical and subcritical) conjugate states of axisymmetric swirling flows being much the same in principle as the hydraulic jump in open-channel flow. Leibovich[31] points out that solutions of (16) cannot give much information like the onset or the position of breakdown, or breakdown and transition in the spiral and bubble forms.

Quasi-cylindrical approximations to (1)-(4) have been introduced by Hall[17,18] and Stewartson and Hall[48]. Vortex breakdown was assumed to be similar to the separation of a two-dimensional boundary layer. They used the quasi-cylindrical equations in step-by-step calculations. Gartshore[14,15] and Mager[37] solved the equations by momentum integral methods by neglecting the imposed pressure gradient. When a large axial gradient develops in their solutions, they conclude that the vortex breakdown has occurred. Just as the two-dimensional boundary-layer approximations break down as soon as the separation occurs, similarly the quasi-cylindrical approximation are no longer true the moment infinitely large axial gradients appear. Mager finds that the closed-form transcendental solutions of the quasi-cylindrical momentum-integral equations for the flow in the viscous core of a wing-tip vortex have two separate branches with the same flow force deficiency. Upstream of the discontinuity, the upper branch solution results in a strongly decelerating flow with a rapidly expanding core while the lower branch solution (with the same angular momentum) gives accelerating flow with a substantially larger but slightly contracting core. These facts together with Sarpkaya's[42] photographs and observations that the axisymmetric bubble is always followed by the spiral breakdown, suggest that the spiral breakdown may be the physical manifestation of the discontinuity while the axisymmetric bubble may be related to the cross-over. However, in words of Leibovich[31], the inference of structural detail from an analysis containing no structure, seems an unusually bold step.

Three sets of investigators attempted to solve numerically the full Navier-Stokes equations. They assumed the flow to be axisymmetric and incompressible. Lavan, Nielsen and Fejer[53] studied the swirling motion in a circular duct, consisting of two smoothly joined sections, (one stationary and the other rotating with a constant angular velocity) for small and large values of the Reynolds numbers. The flow reversal occurs on the axis and near the tube wall and conditions for incipient flow reversal are established. This study deals with a situation much different from those in vortex breakdown. Kopecky and Torrance[26] treated a more realistic problem and imposed initial conditions that resemble vortex breakdown experiments in a tube more closely. An explicit finite difference procedure is used to integrate time dependent transport equations. Their results indicate (a) completely confined eddies can exist even at low Reynolds numbers, (b) sensitivity of eddies to changes in Reynolds number and swirl and (c) some effects of the upstream boundary condition. Calculations performed by Grabowski and Berger[16] in an unconfined region are more extensive and have greater special resolution than those of Kopecky and Torrance. Their solutions exhibit many of the characteristics of vortex breakdown. Taken together the last two results tell us that the Navier-Stokes equations do indeed entertain solutions resembling the axisymmetric bubble form of the vortex breakdown.

SECTION IV

HYDRODYNAMIC INSTABILITY

Numerous authors have investigated the stability of plane parallel flows for many years; however, stability of parallel flows with respect to longitudinal vortex disturbances and transverse wave disturbances in cylindrical coordinates has only received scant attention. In case of zero axial flow Rayleigh[54] derived on an energy consideration the inviscid criterion for the stability of axisymmetric flows that $[d\,(r^2v^2)/dr] > 0$. Later Synge[55] showed that the "Rayleigh Criterion" $[(d/dr)\,(rv)^2] > 0$ is sufficient for stability even with viscosity. Some general stability criteria for nondissipative swirling flows were derived by Howard and Gupta[22]. They showed that an analogy between a rotating and a stratified fluid exists for the stability analysis and that an important determining parameter of stability is a 'Richardson number' based on the analogue of the density gradient and the shear in the axial flow. Batchelor and Gill[2] presented a detailed analytical treatment for the inviscid instability of free axisymmetric flow, in particular jets, and obtained the range of various parameters for which the flow can be unstable. The stability of a potential vortex with a rotating and a non-rotating jet core was analyzed by Lessen, Deshpande and Hadji-Ohanes[56]. They calculated eigen values for different values of the ratio of the strength of the vortex to the axial velocity and showed that the potential vortex becomes unstable in the presence of a jet. Uberoi, Chow and Narain[51] presented the stability analysis of coaxial rotating jet and vortex with different densities and obtained dispersion relation covering a wide range of configurations.

The tip vortex of a laminar flow wing was studied by Singh and Uberoi[45] at a sectional lift-to-drag ratio of 60. Downstream of the wing the jet rapidly dissipated and a wake developed in the core and intensity of turbulent vortices decreased. From 13 to 40 chord length periodic oscillations dominated the velocity fluctuations with little background influence. These instabilities had a symmetric and a helical mode with wavelength of the same order as the core diameter. Garg[13] in his experimental study of the structure of vortex breakdown in a tube observed axial and azimuthal velocity fluctuations at numerous points. It is likely that the oscillations arise from an instability of the mean flow. He measured the mean axial and azimuthal velocity components given by

$$W(r) = W_1 + W_s \exp(-\beta r^2) \tag{17}$$

$$V(r) = \frac{q}{r} [1 - \exp(-\beta r^2)]. \tag{18}$$

The parameters q, β, and W_s vary slowly with axial distance. Their values have been experimentally determined by Garg (see also Leibovich[31]). Figures 3 and 4 show plots of W and V for various values of β, versus r. The stability of the mean velocity profiles given by (17) and (18) has been analyzed by Howard and Gupta[22], Lessen, Singh and Paillet[32] and Lessen and Paillet[33]. To perform the linear stability analysis, equations (1) to (4) are nondimensionalized with respect to the length scale r_s and the velocity scale W_s given by

$$r_s = (W/4\nu z)^{\frac{1}{2}}\ , \quad W_s = \frac{c\bar{o}^2}{8\nu z} \log \frac{Wz}{\nu} + L \frac{W^2}{8\nu z} \tag{19}$$

If we neglect the terms $f(\zeta)$ in equation (12) (Lessen, Singh and Paillet[32] point out that this term is very small under certain conditions), equations (11) and (12) are similar to (17) and (18) when $\beta = \frac{W}{4\nu z}$.

We assume

$$u = u', \quad v = V + v', \quad w = W + w', \quad p = \bar{P} + p' \tag{20}$$

and

$$\{u', v', w', p'\} = \{iG, H, F, P\}\,(r)\exp[i(\alpha z - \alpha ct) + ni\theta] \tag{21}$$

where $\bar{P}$ is the mean pressure distribution given by (13) associated with mean velocity distribution W and V. α and n are axial and azimuthal wave numbers, $c = c_r + ic_i$ is the complex phase velocity and F, G, H and P are the complex amplitudes of perturbation. By substituting (20) and (21) into the nondimensionalized and linearized equations (1) to (4), we get (see Lessen and Paillet[33])

$$\alpha rF + (rG)' + nH = 0 \tag{21}$$

$$r^2\gamma G + 2rVH - r^2P' = (iR)^{-1}\,[r(rG')' - (\alpha^2 r^2 + n^2 + 1)\,G - 2nH] \tag{22}$$

$$r^2\gamma H + r(rV)'G + nrP = (iR)^{-1}[r(rH')' - (\alpha^2 r^2 + n^2 + 1)\,H - 2nG] \tag{23}$$

$$r^2\gamma F + r^2W'G + \alpha^2r^2P = (iR)^{-1}\,[r(rF')' - (\alpha^2 r^2 + n^2)F] \tag{24}$$

where a prime denotes d/dr, $R = r_s\, W_s/\nu$ and $\gamma = \alpha(W - c) + nV/r$.

Batchelor and Gill[2] discuss the boundary conditions required to integrate (21) to (24). These are

$$\begin{aligned} &F(0) = G(0) = H(0) = P(0) = 0 \quad \text{when} \quad n = 0, \neq 1, \\ &F(0) = G(0) = H(0) = P(0) = 0 \quad \text{when} \quad n = 1, 2, 3\ldots\text{(integer)} \end{aligned} \tag{25}$$

$$F(\infty) = F(\infty) = H(\infty) = P(\infty) = 0 \quad \text{for all n.} \tag{26}$$

For the axisymmetric case n = 0, certain general results similar to those of Rayleigh's theorems for the stability of two-dimensional parallel inviscid flows could be established as suggested by Howard and Gupta[22]. Equations (21) to (24) when n = 0 reduce to

$$\frac{d}{dr}\left(\frac{d}{dr} + \frac{1}{r}\right)G = \left[\alpha^2 + \frac{rW'' - W'}{r(W - c)} - \frac{2V(rV' + V)}{r^2(W - c)^2}\right]G \tag{27}$$

Let

$$X = G/(W - c)^{1/2}\,, \quad \phi = r^{-3}\,D(r^2V^2)$$

$$D = d/dr\,, \quad D_* = d/dr + 1/r,$$

then (27) becomes

$$D[(W - c)\,D_*X] + \tfrac{1}{2}\left(\frac{W'}{r} - W''\right)X - \tfrac{1}{4}W'^2\,(W - c)^{-1}X$$

$$- \alpha^2(W - c)X + \phi(W - c)^{-1}X = 0 \qquad (28)$$

Multiplying (28) by rX^* and integrating over (r_1, r_2), we obtain

$$\int_{r_1}^{r_2} (W - c)[|D_* X|^2 + \alpha^2 |X|^2]\, rdr + \tfrac{1}{2}\int_{r_1}^{r_2} (rW'' - W')|X|^2\, rdr$$

$$+ \int_{r_1}^{r_2} (W - c^*)[\tfrac{1}{4}W'^2 - \phi] \left|\frac{X}{W - c}\right|^2 rdr = 0 \qquad (29)$$

Starred quantities denote their corresponding complex conjugate. The imaginary part of (29), if $c_i \equiv I_m c > 0$, gives

$$\int_{r_1}^{r_2} [|D_*X|^2 + \alpha^2 |X|^2]\, rdr + \int_{r_1}^{r_2} [\phi - \frac{1}{4} W'^2] \left|\frac{X}{W - c}\right|^2 rdr = 0 \qquad (30)$$

which is impossible if ϕ is everywhere $\geq \tfrac{1}{4}W'^2$. Thus, a sufficient condition for stability is that $\phi - \tfrac{1}{4}W'^2$ be everywhere non-negative. Defining a local Richardson number $J(r)$ by (Figure 5 shows the profiles of J vs r for various values of β)

$$J(r) = \frac{(r^2V^2)'}{r^3W'^2} \qquad (31)$$

the stability condition is $J(r) \geq \tfrac{1}{4}$. This is similar to Rayleigh's point of inflexion theorem for two-dimensional parallel-flow instability. Another result like that of Howard's circle theorem can be established following Kochar and Jain[25]. If we introduce

$$M = G/(W - c)$$

into (27), we have

$$D[(W - c)^2\, D_*M] - \alpha^2(W - c)^2\, M + \phi M = 0 \qquad (32)$$

If (32) is multiplied by M^*, complex conjugate of M and integrated over (r_1, r_2), then the real and imaginary parts together with certain manipulations introduced by Howard lead the inequality to

$$[(c_r - \frac{a+b}{2})^2 + c_i^2 - (\frac{a-b}{2})^2] \int_{r_1}^{r_2} \rho Q \, rdr + \int_{r_1}^{r_2} \phi |M|^2 \, rdr \leq 0 \tag{33}$$

where $Q = |M'|^2 + \alpha^2 |M|^2$, $a \leq W \leq b$.

The result (33) can be shown such that the complex wave velocity for any unstable mode lies in a semi-ellipse whose major axis coincides with the diameter of Howard's semi-circle while its minor axis depends on the Richardson number J. The following inequality

$$[c_r - \tfrac{1}{2}(a+b)]^2 + \frac{2c_i^2}{1 + (1-4J)^{\frac{1}{2}}} \leq \tfrac{1}{4}(a-b)^2 \tag{34}$$

is satisfied.

The inviscid stability of swirling flows with mean velocity profiles given by (17) and (18) was studied by Lessen, Singh and Paillet with respect to infinitesimal non-axisymmetric disturbances. The flow is characterized by a swirl parameter q which is the ratio of the magnitude of the maximum swirl velocity to that of the maximum axial velocity. It is found that as the swirl is continuously increased from zero, the disturbances die out quickly for a small value of q if n = 1. The results for negative azimuthal modes are very different. For negative values of n, the amplification rate increases and then decreases, falling to negative values at q slightly greater than 1.5 for n = -1. The maximum amplification rate increases for increasingly negative n up to n = -6 (the highest mode investigated) and corresponds to $q \simeq 0.85$. For viscous stability theory, Lessen and Paillet calculated both time wise and space wise growth rates for the lowest three negative non-axisymmetric modes (n = -1, -2, and -3). The large wave numbers associated with the disturbances at large $|n|$ allow the n = -1 mode to have minimum critical Reynolds number of 16 ($q \simeq 0.60$). The other two modes investigated have minimum Reynolds numbers on the neutral curve of 31 ($n = -2$, $q = 0.60$) and 57 ($n = -3$, $q = 0.80$). For each mode, the neutral stability curve is shown to shift rapidly towards infinite Reynolds numbers once the swirl becomes sufficiently large.

SECTION V

CONCLUSION

All the experimental observations indicate that vortex breakdown depend on two dimensionaless numbers (1) the Reynolds $W_s\ r_s/\nu$ and (2) the circulation $\Omega = q/W_s\ r_s$. The axial and swirl velocity components are given by (17) and (18). Starting with these values for the velocity components at a junction $z = 0$, theoretical prediction of the occurrence of axisymmetric 'bubble' like vortex breakdown for $z > 0$ is in qualitative agreement with experimental observations. Linear stability analysis predicts that all the vortex flows are stable subject to infinitesimal axisymmetric disturbances provided the Richardson number $J = \frac{(r^2v^2)'}{r^3W'^2} \geq \frac{1}{4}$. Breakdown has also been correlated with $\tan\phi = (v_{max}/W)$ such that the maximum value of ϕ upstream of breakdown is invaribly greater than about 45°. In the case of non-axisymmetric disturbances, the negative azimuthal modes are more unstable than the positive ones.

Numerical experiments have, however, not been able to predict the spiral form of the breakdown, because it may be excluded by hypothesis by axisymmetric formulation. The effect of adverse pressure gradient on the vortex breakdown has also not been studied.

SECTION VI

RECOMMENDATION

Suggestions for follow-up research: We would like to investigate what effect the adverse pressure gradient has on the vortex breakdown. In flows through pipes both Kirkpatrick[24] and Sarpkaya[42] have observed that there is a slight positive pressure gradient upstream of the breakdown and a negative gradient immediately downstream. In flows past ogive cylinders and delta wings at increasing angle of attack the positive pressure gradient seems to accelerate the vortex breakdown. To study this, first the expressions for axial and azimuthal velocity components similar to (17) and (18) have to be obtained as solutions of the Navier-Stokes equations under the imposed pressure gradient. One way is to invoke (16) with (18) and find ψ for given pressure distributions. Once the questions regarding starting values of v and w are decided, then both the solutions of the Navier-Stokes can be attempted and the linear stability analysis subject to axisymmetric and non-axisymmetric disturbances also can be studied.

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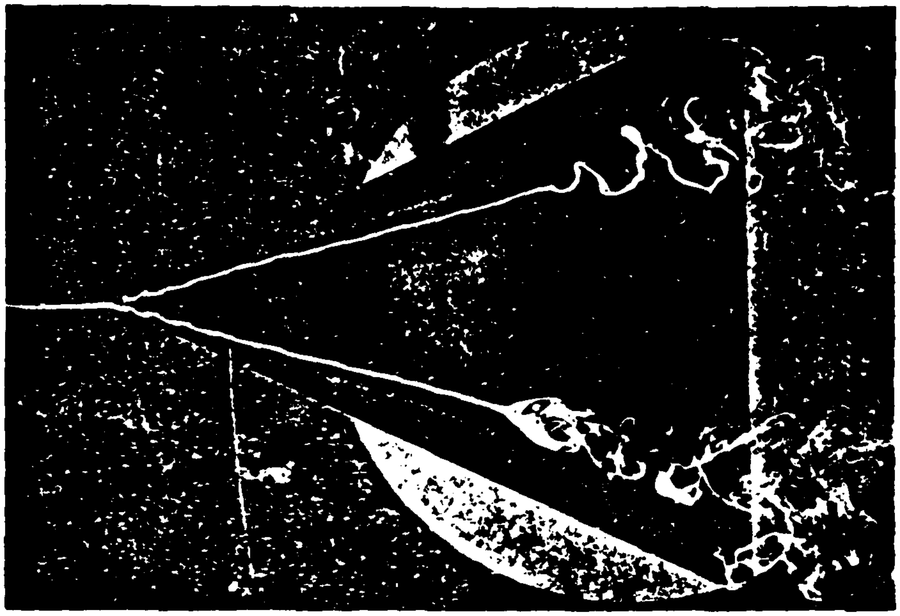

Figure 1. Vortex breakdown over a delta wing (Lambourne & Bryer 28).

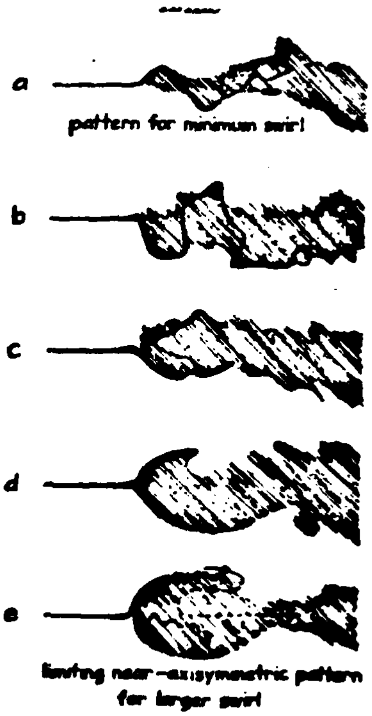

Figure 2. Variation of breakdown pattern with increase of swirl (based on photographs of Sarpkaya 42).

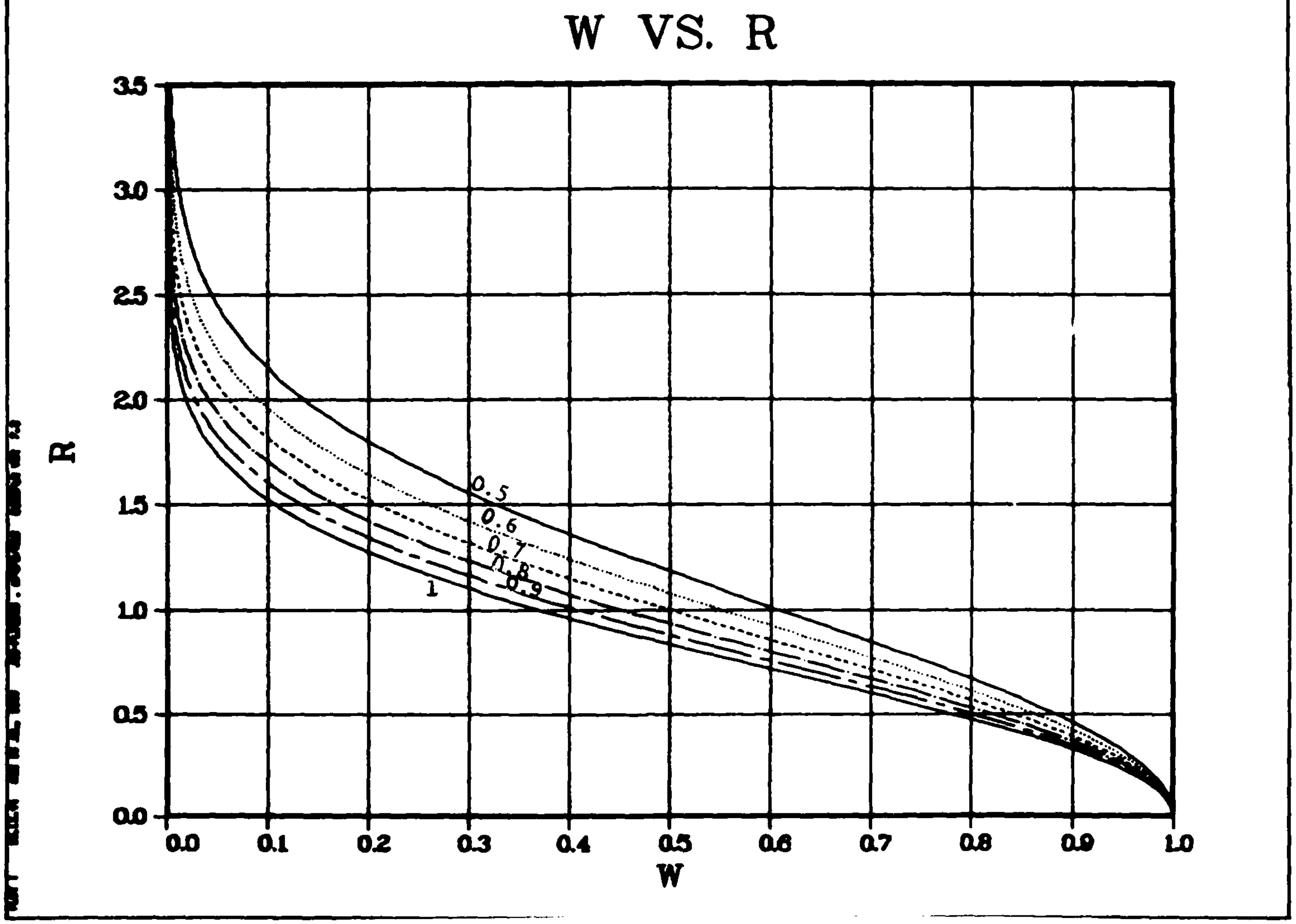

Fig. 3 The axial mean velocity profile W for various values of β.

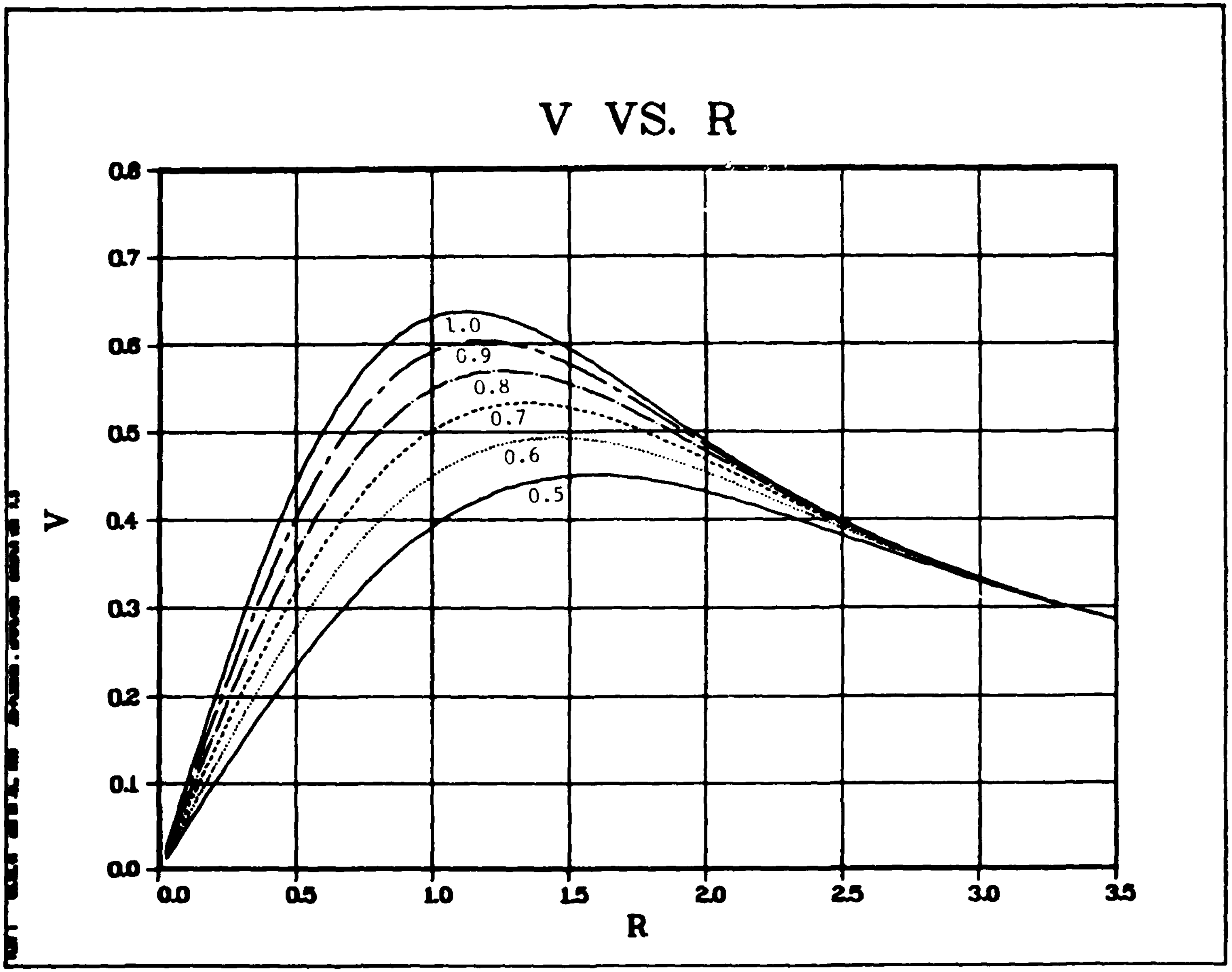

Fig. 4. The azimuthal mean velocity profile V for various values of β.

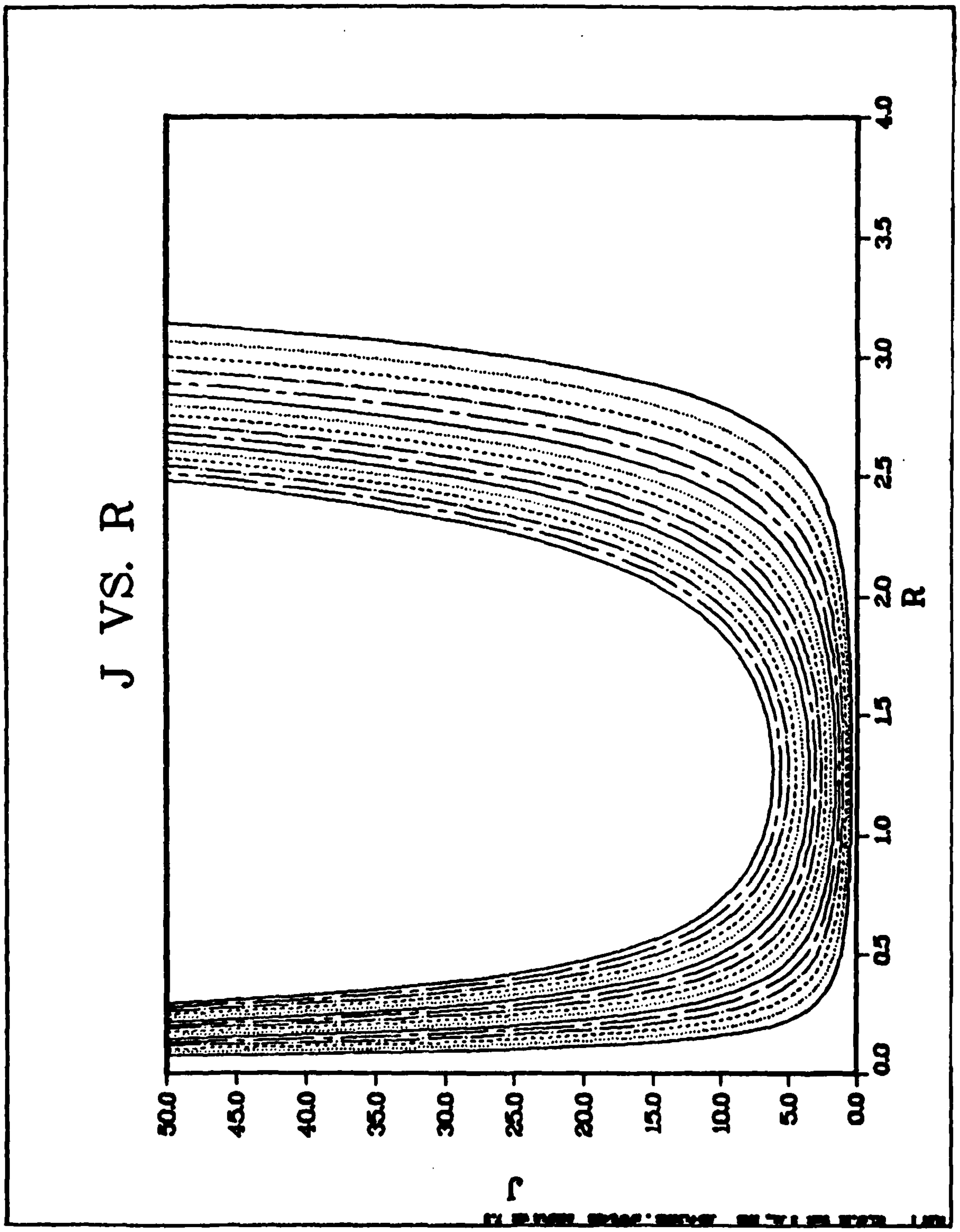

Fig. 5. Profile of Richardson number J versus R for various values of β.

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